1.5 Focus on Selecting Tools and Computational Strategies

When solving problems, it is important to select appropriate tools. You may be able to solve some problems using pencil and paper, while for others you may need a calculator. Sometimes it helps to use manipulatives such as linking cubes or integer chips, while other situations are best tackled using a graph. Quite often, you may find you need several tools. In other situations, you might start with one tool, then find you need another tool to help find the answer.

Investigate

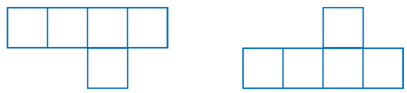
Tools
- square tiles
- linking squares
- grid paper

pentomino
- a shape made of five unit squares
- each square shares at least one side with another square

Which tool(s) can you use to solve a problem?

A: Create shapes with square tiles

Five square tiles are to be used to make different shapes. Sides must align exactly. These shapes are called **pentominos**. Shapes that can be rotated or flipped to form each other are considered the same. For example, the following two are considered the same.



1. a) Make as many different pentominos as you can.
   b) Compare your pentominos with a classmate's.
   c) How many different pentominos are possible?
2. **Reflect** Which tool(s) did you use? Were they effective? Explain.

B: Sums of cubes

Select an appropriate tool to solve the following problem.

1. a) Find the sum of the cubes of the first two natural numbers: $1^3 + 2^3 = ?$
   b) Find the sum of the cubes of the first three natural numbers: $1^3 + 2^3 + 3^3 = ?$
   c) Continue extending these sums and investigate the pattern in the results. Describe the pattern in your own words.
2. a) **Reflect** Verify that your pattern is correct.
   b) Use your pattern to find the sum of the cubes of the first 15 natural numbers.
3. **Reflect** Which tool(s) did you use? Were they effective? Explain.

Example Computational Strategy, Operations With Rational Numbers

rational numbers

- numbers that can be expressed as the quotient of two integers, where the divisor is not zero
- $\frac{3}{5}$, 0.25, $-1\frac{3}{4}$, and -3 are rational numbers

Add or subtract each pair of **rational numbers**, as indicated.

a) $\frac{3}{8}+\left(-\frac{1}{8}\right)$ **b)** $-\frac{1}{2}+\left(-\frac{2}{3}\right)$ **c)** $-\frac{3}{5}-\left(-\frac{1}{4}\right)$

Solution

The strategy for adding and subtracting rational numbers is to connect your skills with fractions and integers.

a)
$$\frac{3}{8}+\left(-\frac{1}{8}\right)$$

The denominators are the same, so I can add the numerators. 3 + (−1) = 2

$$= \frac{2}{8}$$
$$= \frac{1}{4}$$

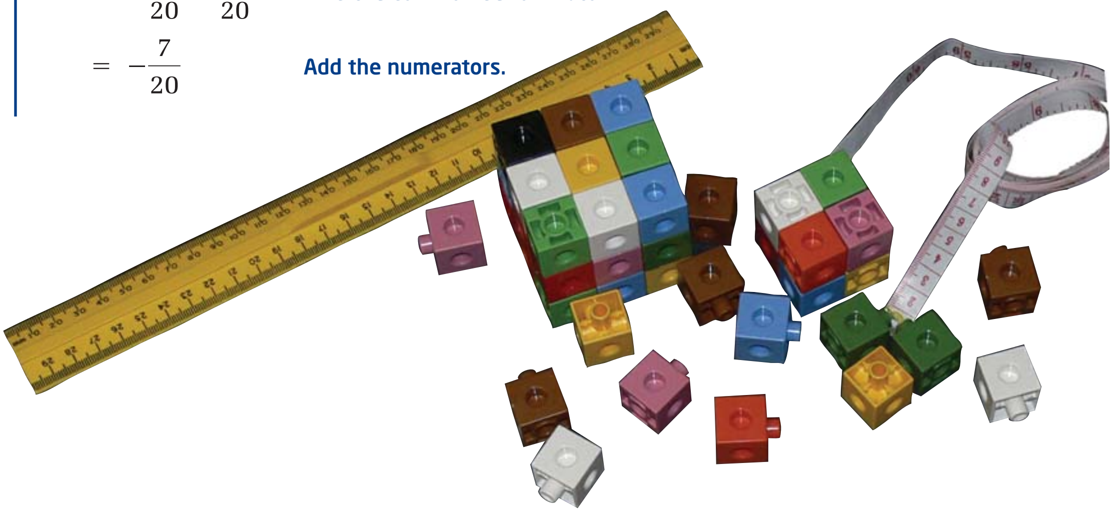
This fraction isn't in lowest terms. I can divide the numerator and the denominator by 2.

b)
$$-\frac{1}{2}+\left(-\frac{2}{3}\right)$$

The denominators are different, so I need to find a common denominator.

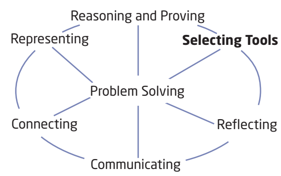
$$= -\frac{3}{6}+\left(-\frac{4}{6}\right)$$
$$= -\frac{7}{6}$$ Add the numerators.
$$= -1\frac{1}{6}$$ Change to a mixed number.

c)
$$-\frac{3}{5}-\left(-\frac{1}{4}\right)$$
$$= -\frac{3}{5}+\frac{1}{4}$$ Change subtracting to adding the opposite.
$$= -\frac{12}{20}+\frac{5}{20}$$ Find the common denominator.
$$= -\frac{7}{20}$$ Add the numerators.



Key Concepts

- Tools such as calculators, physical models, graph paper, and computers can help you solve problems.
- Choosing the best tool for a given situation can make you a more efficient problem solver. For example,
  - adding 10 and 20 on a scientific calculator would take longer than finding the sum mentally
  - using a 30-cm ruler to measure the length of a soccer field would take longer than using a trundle wheel
- A variety of computational strategies need to be considered when investigating mathematical ideas and solving problems.



Communicate Your Understanding

C1 A Fermi problem asks how many times a truck wheel turns in driving along the 401 highway from Windsor to London. What tools would you use to solve this problem?

C2 Ted used a calculator to evaluate the expression $\frac{5}{9} + \frac{2}{3} - \frac{1}{8}$ in the following way:

$$\frac{5}{9} + \frac{2}{3} - \frac{1}{8}$$
$$= 0.6 + 0.7 - 0.1$$
$$= 1.2$$

Explain what Ted's error was and how he could have used his calculator more appropriately.

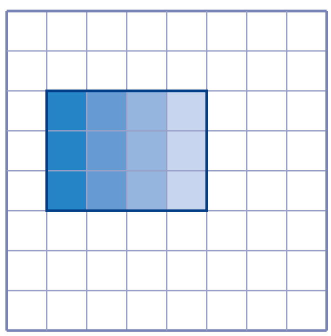
Practise

1. a) Explain how the diagram illustrates the fact that $12 \div 4 = 3$.

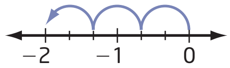
b) Draw a diagram to illustrate that $12 \div 3 = 4$.

c) Draw a diagram to illustrate that $12 \div 2 = 6$.

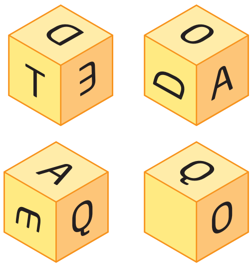
d) Continue the pattern. How do these models show that $12 \div 0$ is not defined?



2. a) Explain how the diagram illustrates the fact that $3 \times \left(-\frac{2}{3}\right) = -2$



b) Use a visual tool to model $4 \times \left(-1\frac{1}{4}\right)$.



3. a) Four views of a cube are shown. What letter belongs on the blank face, and which way should the letter face?

b) What tool did you use to help solve this problem? Was it effective?

c) Make up your own similar problem, using numbers instead of letters. Have a classmate solve the problem.

4. What tools would you use to find the average cost of admission to a movie in your area?

5. Describe when it would be appropriate to use each tool to solve a mathematical problem. Give examples.

a) a calculator

b) grid paper

c) a physical model

d) a computer

6. The number 90 224 199 is the fifth power of what number? Which tool did you use?

7. Use an appropriate tool and strategy to find the two missing values in each sequence.

a) 15, 9, 3, … , ■, ■, −69

b) 5, 15, 45, … , ■, ■, 10 935

c) −1024, 512, −256, … , ■, ■, −1

d) −5, −8, −11, … , ■, ■, −164

e) 3, −6, 12, … , ■, ■, −24 576

f) −400, −376, −352, … , ■, ■, 80

8. Use appropriate tools and strategies to find the next three terms in each sequence.

a) 240, 120, 40, 10, 2

b) $0, -\frac{1}{3}, -\frac{2}{3}, -1$

c) $\frac{3}{4}, \frac{1}{2}, \frac{1}{4}$

d) $\frac{2}{3}, \frac{7}{12}, \frac{1}{2}, \frac{5}{12}$

9. Find each sum.

a) $-\frac{1}{2} + \left(-\frac{1}{2}\right)$

b) $-\frac{2}{3} + \left(-\frac{3}{4}\right)$

c) $\frac{1}{7} + \left(-\frac{2}{5}\right)$

d) $-\frac{2}{3} + \frac{3}{8}$

10. Find each difference.

a) $\frac{3}{8} - \frac{5}{6}$

b) $\frac{1}{2} - \frac{2}{3}$

c) $\left(-\frac{1}{4}\right) - \frac{1}{6}$

d) $\left(-\frac{4}{5}\right) - \left(-\frac{3}{10}\right)$

Connect and Apply

11. The Example demonstrated how to add and subtract rational numbers. Describe how to multiply and divide rational numbers. Provide examples.

12. Evaluate.

a) $-\frac{5}{6} \times \frac{3}{10}$

b) $\left(-\frac{1}{7}\right) \times \left(-\frac{3}{5}\right)$

c) $\left(-\frac{1}{8}\right) \times \frac{6}{11}$

d) $\frac{7}{8} \div \left(-\frac{5}{6}\right)$

e) $\left(-\frac{5}{12}\right) \div \left(-\frac{3}{8}\right)$

f) $\left(-4\frac{2}{5}\right) \div 1\frac{4}{7}$

13. A sheet of paper is 0.08 mm thick.

a) How many times do you think you can fold a sheet of paper in half?

b) If you fold it in half, how thick are the two layers?

c) If you could fold it in half again and again, a total of 20 times, how thick would the layers be? Are you surprised at the answer? Why?

d) Find out for yourself how many times you can fold a piece of paper in half. Explain the results.

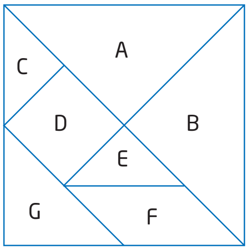
14. Explain how you would use two different tools to help a younger student understand how to add each pair of fractions.

a) $\frac{1}{2} + \frac{1}{4}$

b) $\frac{2}{3} + \frac{4}{5}$

15. Use a geoboard or centimetre dot paper to find how many different rectangles with a perimeter of 20 cm and whole-number side lengths can be made. Find the area of each rectangle. Record your results in a table.

Length (cm)	Width (cm)	Perimeter (cm)	Area (cm^2)
		20	



16. This is a tangram, which is a very old puzzle that originated in ancient China.

a) Determine the fraction of the whole square that each labelled shape represents.

b) What fraction of the whole square does each of the following represent? Illustrate your answer using pieces of the tangram and using operations with fractions.

i) A + B **ii)** C + G **iii)** D + E

iv) F − E **v)** $\frac{1}{4}$A **vi)** $\frac{1}{2}$D − F

17. Use the tangram in question 16.

a) Write piece F as the sum of two or more smaller pieces.

b) Write piece B as the sum of two or more smaller pieces.

18. Use an appropriate tool to help determine the thousandth term in the sequence 45, 41, 37, 33, … .

19. Use an appropriate tool to help determine which term in the sequence 100, 93, 86, … is −600.

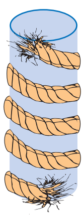
20. How many cups of water would fill up a bathtub? Explain your reasoning.

Extend

21. If you fold a piece of string in half, in half again, and so on, up to *n* folds, and then cut it through the middle with a pair of scissors, how many pieces of string will you have?

a) Develop a solution. Explain your reasoning.

b) Use a model to verify your solution.



22. A rope winds around a cylindrical tube a total of four times. The tube has a circumference of 10 m and a height of 24 m. How long is the rope?